Last week we promised you a couple of tricky standard deviation (SD) GMAT questions. We start with a 600-700 level question and then look at a 700 – 800 level one.

Question 1: During an experiment, some water was removed from each of the 8 water tanks. If the standard deviation of the volumes of water in the tanks at the beginning of the experiment was 20 gallons, what was the standard deviation of the volumes of water in the tanks at the end of the experiment?

Statement 1: For each tank, 40% of the volume of water that was in the tank at the beginning of the experiment was removed during the experiment.

Statement 2: The average volume of water in the tanks at the end of the experiment was 80 gallons.

Solution:

We have 8 water tanks. This implies that we have 8 elements in the set (volume of water in each of the 8 tanks). SD of the volume of water in the tanks is 20 gallons. We need to find the new SD i.e. the SD after water was removed from the tanks.

Statement 1: For each tank, 40% of the volume of water that was in the tank at the beginning of the experiment was removed during the experiment.

Initial SD is 20. When 40% of the water is removed from each tank, the leftover water is 60% of the initial volume of water i.e. 0.6*initial volume of water. This means that each element of the initial set was multiplied by 0.6 to obtain the new set. The SD will change. It will become 0.6*previous SD i.e. 0.6*20 = 12 (think of the formula of SD we discussed in the first SD post). This statement alone is sufficient.

Statement 2: The average volume of water in the tanks at the end of the experiment was 80 gallons.

The average volume doesn't give us the SD of the new set. Hence, this statement alone is not sufficient.

Answer (A)

Now that we are done with the easier one, let's go on to the tougher one.

Question 2: M is a collection of four odd integers. The range of set M is 4. How many distinct values can standard deviation of M take?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

Solution:

Since the range of M is 4, it means the greatest difference between any two elements is 4. One way of doing this will be M = {1, x, y, 5} (obviously, there are innumerable ways of writing M)
Here, x and y can take one of 3 different values: 1, 3 and 5 (x and y cannot be less than 1 or greater than 5 because the range of the set is 4).

Both x and y could be same. This can be done in 3 ways. Or x and y could be different. This can be done in 3C2 = 3

ways. Total x and y can take values in 3 + 3 = 6 ways.

(Note here that the number of ways in which you can select x and y is not 3*3 = 9. Why?)

For clarification, let me enumerate the 6 ways in which you can get the desired set:
{1, 1, 1, 5}, {1, 3, 3, 5}, {1, 5, 5, 5}, {1, 1, 3, 5}, {1, 1, 5, 5}, {1, 3, 5, 5}

Note here that standard deviations of {1, 1, 1, 5} and {1, 5, 5, 5} are same. Why? Because SD measures deviation from mean. It has nothing to do with the actual value of mean and actual value of numbers.

Mean of {1, 1, 1, 5} is 2. Three of the numbers are distance 1 away from mean and one number is distance 3 away from mean. Mean of {1, 5, 5, 5} is 4. Three of the numbers are distance 1 away from mean and one number is distance 3 away from mean. Sum of the squared deviations will be the same in both the cases and the number of elements is also the same in both the cases. Therefore, both these sets will have the same SD.

Similarly, {1, 1, 3, 5} and {1, 3, 5, 5} will have the same SD.

From the leftover sets, {1, 3, 3, 5} will have a distinct SD and {1, 1, 5, 5} will have a distinct SD.

In all, there are 4 different values that SD can take in such a case.

Note: It doesn't matter what the actual numbers are. Since we have found 4 distinct values for SD, we will always have 4 distinct values of SD for a set under the given constraints.

Answer (B)

Hope the question was fun for you too!